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Preface

This textbook is written for advanced-placement freshman physics concentrators. The content consists partly of text and partly of problems with detailed solutions. The latter is what I think will make this book more useful than others. If you are searching for a supply of practice problems to work on, this is it.

A brief outline of the book is as follows. Chapter 1 covers statics and is an ‘extra’ chapter. It is mainly review, and most of it should look familiar; but it has lots of fun problems. In Chapter 2, we learn about forces and how to apply $F = ma$. There’s a bit of math here in solving some simple differential equations. Chapter 3 deals with oscillations and coupled oscillators. Again, there’s a fair amount of math here in solving linear differential equations, but there’s no way to avoid it. Chapter 4 deals with conservation of energy and momentum. You’ve probably seen much of this before.

In Chapter 5, we introduce the Lagrangian method, which will undoubtedly be new to you. It looks rather formidable at first, but it’s really not all that rough. There are difficult concepts at the heart of the subject, but the nice thing is that the technique is easy to apply. The situation here is similar to taking a derivative in calculus; there are substantive concepts on which the theory rests, but the act of taking a derivative is fairly mindless.

Chapter 6 deals with central forces, Kepler’s Laws, and such things. Chapter 7 covers the easier type of angular momentum problems, ones where the direction of the angular momentum is fixed. Chapter 8 covers the more difficult type, ones where the direction changes. Gyroscopes, spinning tops, and other fun and perplexing objects fall into this category. Chapter 9 deals with accelerated frames of reference and fictitious forces.

Chapters 10 through 13 cover relativity. Chapter 10 deals with relativistic kinematics — abstract particles flying through space and time. In Chapter 11, we discuss relativistic dynamics — energy, momentum, force, etc. Chapter 12 introduces the important concept of ‘4-vectors’. The material in this chapter could alternatively be put in the previous two, but for various reasons I thought it best to create a separate chapter for it. Chapter 13 covers a few topics from general relativity. There is no possible way that one chapter can do this subject justice, of course, so we’ll just look at some basic examples.

The appendices contain various useful things. Indeed, Appendices B and C are the first parts of this book you should read.

Throughout the book, I have included many ‘remarks’. These are written in a

slightly smaller font than the surrounding text. They begin with a small-capital ‘REMARK’, and they end with a shamrock (♣). The purpose of these remarks is to say something that needs to be said, without disrupting the overall flow of the argument. In some sense these are ‘extra’ thoughts, but they are invariably useful in understanding what is going on. They are usually more informal than the rest of the text. I reserve the right to occasionally use them to babble about things I find interesting, but which you may find a bit tangential. For the most part, however, the remarks address issues and questions that arise naturally in the course of the discussion.

At the end of the solutions to many problems, the obvious thing to do is to check limiting cases.¹ I have written these in a smaller font, but I have not always bothered to start them with a ‘REMARK’ and end them with a ‘♣’, because they are not ‘extra’ thoughts; checking limiting cases of your answer is something you should *always* do.

For your reading pleasure (I hope), I have included a few limericks scattered throughout the text. At best, they are educational. At worst, they will lighten things up. Some are funny. Some are stupid. But at least they are all physically accurate (give or take).

A word on the problems. Some are easy, but many are quite difficult. I think you will find them quite interesting, *but do not get discouraged if you have trouble solving them. Some are designed to be brooded over for hours.* They exist for your enjoyment only; you are not required to do any. If you don’t touch any of them, but if you exert a large effort on the homework problems and all the other required work, then you should do well in this course. I have chosen to write them up for two reasons: (1) Students invariably want extra practice problems, with solutions, to work on, and (2) I find them rather fun.

I have written up solutions for the vast majority of the problems. The ones with solutions are labeled ‘Problems’, and the ones without solutions are labeled ‘Exercises’. The exercises come right before the problems at the end of each chapter.

For the problems you do choose to work on, be careful not to look at the solution too soon. There is nothing wrong with putting a problem aside for a while and coming back to it later. Indeed, this is probably the best way to approach one. If you head to the solution at the first sign of not being able to solve a problem, then you have wasted the problem.

REMARK: This gives me an opportunity for my first remark (and first limerick, too). One thing many people don’t realize is that you need to know more than the correct way(s) to do a problem; you also need to be familiar with a lot of *incorrect* ways to do it. Otherwise, when you come upon a new problem, there may be a number of decent-looking approaches to take, and you won’t be able to immediately weed out the poor ones. Struggling a bit with a problem invariably leads you down some wrong paths, and this is an essential part of learning. To understand something, you not only have to know what’s right about the right things; you also have to know what’s wrong about the wrong things. Learning takes a serious amount of effort, a few wrong turns, and a lot of sweat. Alas, there are no short-cuts to understanding physics.

¹This topic is discussed in Appendix C.

The ad said, For one little fee,
You can skip all that course-work ennui.
So send your tuition,
No need for admission!
Get your mail-order physics degree! ♣

The problems are marked with a number of asterisks. Harder problems earn more asterisks (on a scale from zero to four). You may, of course, disagree with my judgment of difficulty. But I think an arbitrary weighting scheme is better than none at all. Enjoy!

— David Morin

